# Chapter 8 – Sampling Distributions

## OUTLINE

1. Distribution of the Sample Mean
2. Distribution of the Sample Proportion

## Putting It Together

In chapters 6 and 7, we learned about random variables and their probability distributions.

In this chapter, we continue our discussion of probability distributions where statistics, such as , will be the random variable. Statistics are random variables because the value of a statistic varies from sample to sample. For this reason, statistics have probability distributions associated with them. For example, there is a probability distribution for the sample mean, sample proportion, and so on. We use probability distributions to make probability statements regarding the statistic. In this chapter, we examine the shape, center, and spread of statistics such as .

## Section 8.1 Distribution of the Sample Mean

### Objectives

1. Describe the Distribution of the Sample Mean: Normal Population
2. Describe the Distribution of the Sample Mean: Non-normal Population

Introduction, Page 1

 *Watch the video for an overview of where we have been and where we are going in the course.*

In Chapters 1 through 4 we learned how to identify the research objective (Step 1 of the statistical process) as well as collect (Step 2) and describe data (Step 3). In Chapters 5 through 7 we developed the skills that allow us to perform inference (Step 4). Because it is difficult to gain access to populations, the data found in Step 2 is often from a sample. Sample data are used to make inferences about the population. For example, we might compute the mean of a sample and use this information to draw conclusions regarding the population mean. The rest of this course focuses on how sample data are used to draw conclusions about populations.

Introduction, Page 2

 *Watch the video for an overview of the material presented in this chapter.*

A random variable is a numerical measure of the outcome of a probability experiment. Statistics such as the sample mean, , are random variables. Statistics are random variables because the value of a statistic varies from sample to sample. For this reason, statistics have probability distributions associated with them. For example, there is a probability distribution for the sample mean and the sample proportion.

Introduction, Page 3

The sample mean will vary from sample to sample. Our goal in this section is to describe the distribution of the sample mean. Remember, when we describe a distribution, we do so in terms of its shape, center, and spread.

Introduction, Page 4

1. What is the sampling distribution of a statistic?

Introduction, Page 4 (Continued)

1. What is the sampling distribution of the sample mean ?
2. List the three steps for determining the sampling distribution of the sample mean.

Once a particular sample is obtained, it cannot be obtained a second time.

#### Objective 1: Describe the Distribution of the Sample Mean: Normal Population

Objective 1, Page 1

*Answer the following after watching the video.*

1. Describe the shape of the distribution of the sample mean as the sample size increases.

Objective 1, Page 1 (Continued)

1. What does the mean of the distribution of the sample mean, *,* equal?
2. As the sample size *n* increases, what happens to the standard deviation of the distribution of the sample mean?

Objective 1, Page 2

1. List the formulas for the mean and standard deviation of the sampling distribution of .
2. What is the standard error of the mean?

Objective 1, Page 3

Note, in both simulations, the standard error of the mean was close to the approximate standard error.

Objective 1, Page 4

1. Describe the shape of the sampling distribution of  if the random variable *X* is normally distributed.

Objective 1, Page 7

**Example 1 *Finding Probabilities of a Sample Mean***

The IQ, *X*, of humans is approximately normally distributed with mean  and standard deviation . Compute the probability that a simple random sample of size *n* = 10 results in a sample mean greater than 110. That is, compute .

#### Objective 2: Describe the Distribution of the Sample Mean: Non-normal Population

Objective 2, Page 1

 *Answer the following after Activity 1: Sampling Distribution of the Sample Mean: Non-normal Population*

1. As the sample size increases, describe the effect on the center and spread of the distribution.

Objective 2, Page 2

 Watch the video to help reinforce the concepts from Activity 1.

Objective 2, Page 3

1. What is the mean of the sampling distribution of the sample mean equal to? What is the standard deviation of the sampling distribution of the sample mean equal to?

Objective 2, Page 3 (continued)

1. What happens to the shape of the sampling distribution of the sample mean as the sample size increases?
2. State the Central Limit Theorem.

Objective 2, Page 4

How large does the sample size need to be before we can say that the sampling distribution of  is approximately normal? The answer depends on the shape of the distribution of the underlying population. Distributions that are highly skewed will require a larger sample size for the distribution of  to become approximately normal.

Objective 2, Page 5

Notice that even for a highly skewed population of household incomes for a town, the distribution of the sample mean is approximately normal for *n* = 25.

Objective 2, Page 6

1. State the rule of thumb for invoking the Central Limit Theorem.

Objective 2, Page 9

**Example 2 *Weight Gain during Pregnancy***

The mean weight gain during pregnancy is 30 pounds, with a standard deviation of 12.9 pounds. Weight gain during pregnancy is skewed right. An obstetrician obtains a random sample of 35 low-income patients and determines that their mean weight gain during pregnancy was 36.2 pounds. Does this result suggest anything unusual?

Objective 2, Page 11

 *Watch the video for a summary of the shape, center, and spread of the distribution of the sample mean for both normal and non-normal populations.*

## Section 8.2 Distribution of the Sample Proportion

### Objectives

1. Describe the Sampling Distribution of a Sample Proportion
2. Compute Probabilities of a Sample Proportion

#### Objective 1: Describe the Sampling Distribution of a Sample Proportion

Objective 1, Page 1

1. Define the sample proportion, .

Objective 1, Page 2

**Example 1 *Computing a Sample Proportion***

The Harris Poll conducted a survey of 1200 adult Americans who vacation during the summer and asked whether the individuals planned to work while on summer vacation. Of those surveyed, 552 indicated that they planned to work while on vacation. Find the sample proportion of individuals surveyed who planned to work while on summer vacation.

Objective 1, Page 5

Because the value of the sample proportion, , varies from sample to sample, it is a random variable and has a probability distribution.

Objective 1, Page 6

 *Answer the following after watching the video.*

1. As the sample size increases, describe what happens to the shape of the sampling distribution of the sample proportion.
2. What does the mean of the sampling distribution of the sample proportion equal?
3. As the sample size increases, describe what happens to the standard deviation of the sampling distribution of the sample proportion.

Objective 1, Page 7

 Answer the following after Activity 1: Sampling Distribution of the Sample Proportion.

1. What is the mean of the distribution in all three cases?
2. What role does sample size play in the standard deviation?

Objective 1, Page 7 (Continued)

1. What role does sample size play in the shape of the sampling distribution of ?

Objective 1, Page 8

1. Under what conditions is the shape of the sampling distribution of  approximately normal?
2. State the formulas for the mean and standard deviation of the sampling distribution of .

The sample size, *n*, can be no more than 5% of the population size, *N*. That is, 

Objective 1, Page 10

**Example 2 *Describing the Sampling Distribution of the Sample Proportion***

Based on a study conducted by the Gallup Organization, 77% of Americans believe that the state of moral values in the United States is getting worse. Suppose we obtain a simple random sample of n = 60 Americans and determine which of them believe that the state of moral values in the United States is getting worse. Describe the sampling distribution of the sample proportion for Americans with this belief.

#### Objective 2: Compute Probabilities of a Sample Proportion

Objective 2, Page 1

Now that we know how to describe the sampling distribution of the sample proportion, we can compute probabilities involving sample proportions.

Objective 2, Page 2

**Example 3 *Computing Probabilities of a Sample Proportion***

According to the National Center for Health Statistics, 15% of all Americans have hearing trouble.

1. In a random sample of 120 Americans, what is the probability that at most 12% have hearing trouble?
2. Suppose that a random sample of 120 Americans who regularly listen to music using headphones results in 26 having hearing trouble. What might you conclude?